1022. Proposed by Elias Lampakis, Kiparissia, Greece.

Let a,b,c be the sidelengths opposite angles A,B,C of an acute $\triangle ABC$. Prove that

$$\frac{a\cos(B-C)}{b\cos(C-A)+a\cos A}+\frac{b\cos(C-A)}{c\cos(A-B)+b\cos B}+\frac{c\cos(A-B)}{a\cos(B-C)+c\cos C}\geq 2.$$

Solution by Arkady Alt, San Jose, California, USA.

Since $a\cos(B-C) = R\cdot 2\sin A\cos(B-C) = R(\sin(A+B-C)+\sin(A-B+C)) = R(\sin(180^\circ-2C)+\sin(180^\circ-2B)) = R(\sin 2C+\sin 2B) = 2R(\sin C\cos C+\sin B\cos 2B) = c\cos C+b\cos B$ and, similarly, $b\cos(C-A) = a\cos A+c\cos C$ then

$$\sum_{cyclic}\frac{a\cos(B-C)}{b\cos(C-A)+a\cos A} = \sum_{cyclic}\frac{c\cos C+b\cos B}{2a\cos A+c\cos C}.$$

Let $x := a\cos A, y := b\cos B, z := c\cos C$. Since by Cauchy Inequality

$$\sum_{cyclic}\frac{a\cos(B-C)}{b\cos(C-A)+a\cos A} = \sum_{cyclic}\frac{y+z}{2x+z} = \sum_{cyclic}\frac{(y+z)^2}{(2x+z)(y+z)} \geq \frac{\left(\sum_{cyclic}(y+z)\right)^2}{\sum_{cyclic}(2x+z)(y+z)} =$$

$$\frac{4(x+y+z)^2}{x^2+y^2+z^2+5(xy+yz+zx)} = \frac{4(x+y+z)^2}{(x+y+z)^2+3(xy+yz+zx)},$$ and

$3(xy+yz+zx)\leq(x+y+z)^2$ then $\dfrac{4(x+y+z)^2}{(x+y+z)^2+3(xy+yz+zx)} \geq \dfrac{4(x+y+z)^2}{2(x+y+z)^2} = 2.$